

# Sum rules of electromagnetic response functions in $^{12}\text{C}$

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In collaboration with:

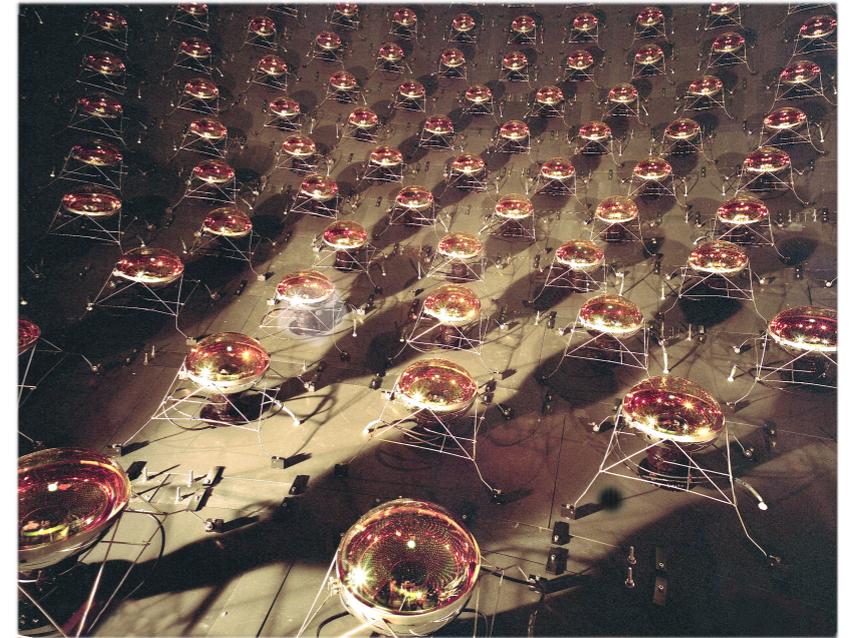
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# Introduction

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- The electroweak response is a fundamental ingredient to describe the neutrino - Carbon-12 scattering, recently measured by the MiniBooNE collaboration to calibrate the detector aimed at studying neutrino oscillations.
  
- As a first step towards its calculation, we have computed the sum rules for the electromagnetic response of  $^{12}\text{C}$ . We want to predict the results of Jefferson lab experiment nearing publication.



# Electromagnetic response

The electromagnetic inclusive cross section of the process

$$e + {}^{12}\text{C} \rightarrow e' + X$$

where the target final state is undetected, can be written in the Born approximation as

$$\frac{d^2\sigma}{d\Omega_{e'} dE_{e'}} = -\frac{\alpha^2}{q^4} \frac{E_{e'}}{E_e} L_{\mu\nu} W^{\mu\nu},$$

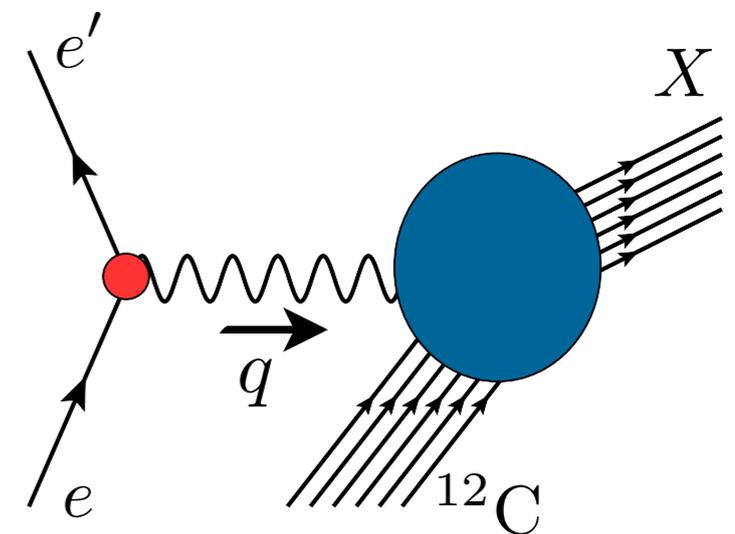
## Leptonic tensor

$$L_{\mu\nu} = 2[k_\mu k'_\nu + k_\nu k'_\mu - g_{\mu\nu}(kk')]$$

## Hadronic tensor

$$W^{\mu\nu} = \sum_X \langle \Psi_0 | J^\mu | \Psi_X \rangle \langle \Psi_X | J^\nu | \Psi_0 \rangle \delta^{(4)}(p_0 + q - p_X)$$

It contains all the information on target structure.



# Electromagnetic response

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- At moderate momentum transfer, non relativistic wave functions can be used to describe the initial and final states and an expansion of the current operator in powers of  $|\mathbf{q}|/m$  can be performed.
- The hadronic tensor (and the cross section) can be written in terms of the longitudinal and transverse response functions, with respect to the direction of the three-momentum transfer:

## Longitudinal

$$R_L(q, \omega) = \sum_X \langle \Psi_0 | \rho | \Psi_X \rangle \langle \Psi_X | \rho | \Psi_0 \rangle \delta(E_0 + \omega - E_X)$$

## Transverse

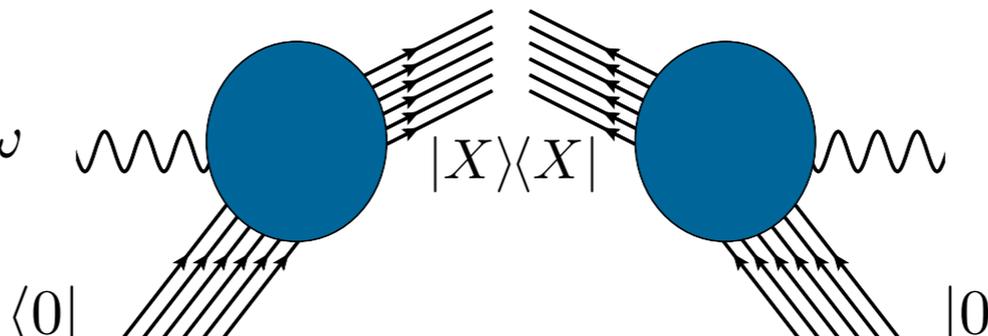
$$R_T(q, \omega) = \sum_X \langle \Psi_0 | \vec{j}_T^\dagger | \Psi_X \rangle \langle \Psi_X | \vec{j}_T | \Psi_0 \rangle \delta(E_0 + \omega - E_X)$$

# Electromagnetic sum rules

- The direct calculation of the response requires the knowledge of all the transition amplitudes:  $\langle \Psi_0 | \rho | \Psi_X \rangle$  and  $\langle \Psi_0 | \vec{j}_T | \Psi_X \rangle$ .
- The sum rules provide an useful tool for studying integral properties of the electron-nucleus scattering.

$$S_\alpha(q) = C_\alpha \int_{\omega_{\text{th}}^+}^{\infty} d\omega \frac{R_\alpha(q, \omega)}{G_E^{p2}(Q^2)} \rightarrow \text{Proton electric form factor}$$

- Using the completeness relation, they can be expressed as ground-state expectation values of the charge and current operators.

$$S_\alpha(q) = \sum_X \int d\omega$$


# Longitudinal and transverse sum rules.

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## Longitudinal sum rule

$$S_L(\mathbf{q}) = C_L \left[ \frac{1}{G_E^p(Q_{qe}^2)} \langle 0 | \rho(\mathbf{q}) \rho(\mathbf{q}) | 0 \rangle - \frac{1}{G_E^p(Q_{el}^2)} |\langle 0; \mathbf{q} | \rho(\mathbf{q}) | 0 \rangle|^2 \right] \quad ; \quad C_L = \frac{1}{Z}$$

The elastic contribution, proportional to the longitudinal form factor has been removed.


$$F_L(\mathbf{q}) = C_L \langle 0; \mathbf{q} | \rho(\mathbf{q}) | 0 \rangle$$

## Transverse sum rule

$$S_T(\mathbf{q}) = \frac{C_T}{G_E^p(Q_{qe}^2)} \langle 0 | \vec{j}_T^\dagger(\mathbf{q}) \vec{j}_T(\mathbf{q}) | 0 \rangle \quad ; \quad C_T = \frac{2}{(Z \mu_p^2 + N \mu_n^2)} \frac{m^2}{q^2}$$

- $C_L$  and  $C_T$  have been introduced under in order for  $S_\alpha(q \rightarrow \infty) \rightarrow 1$  in the approximation where nuclear charge and current operators originate solely from the charge and spin magnetization of individual protons and neutrons and that relativistic corrections are ignored.



# Ab-initio few-nucleon calculation

- The density and current operators have to be consistent with the realistic nucleon-nucleon (NN) interaction, controlled by scattering data.

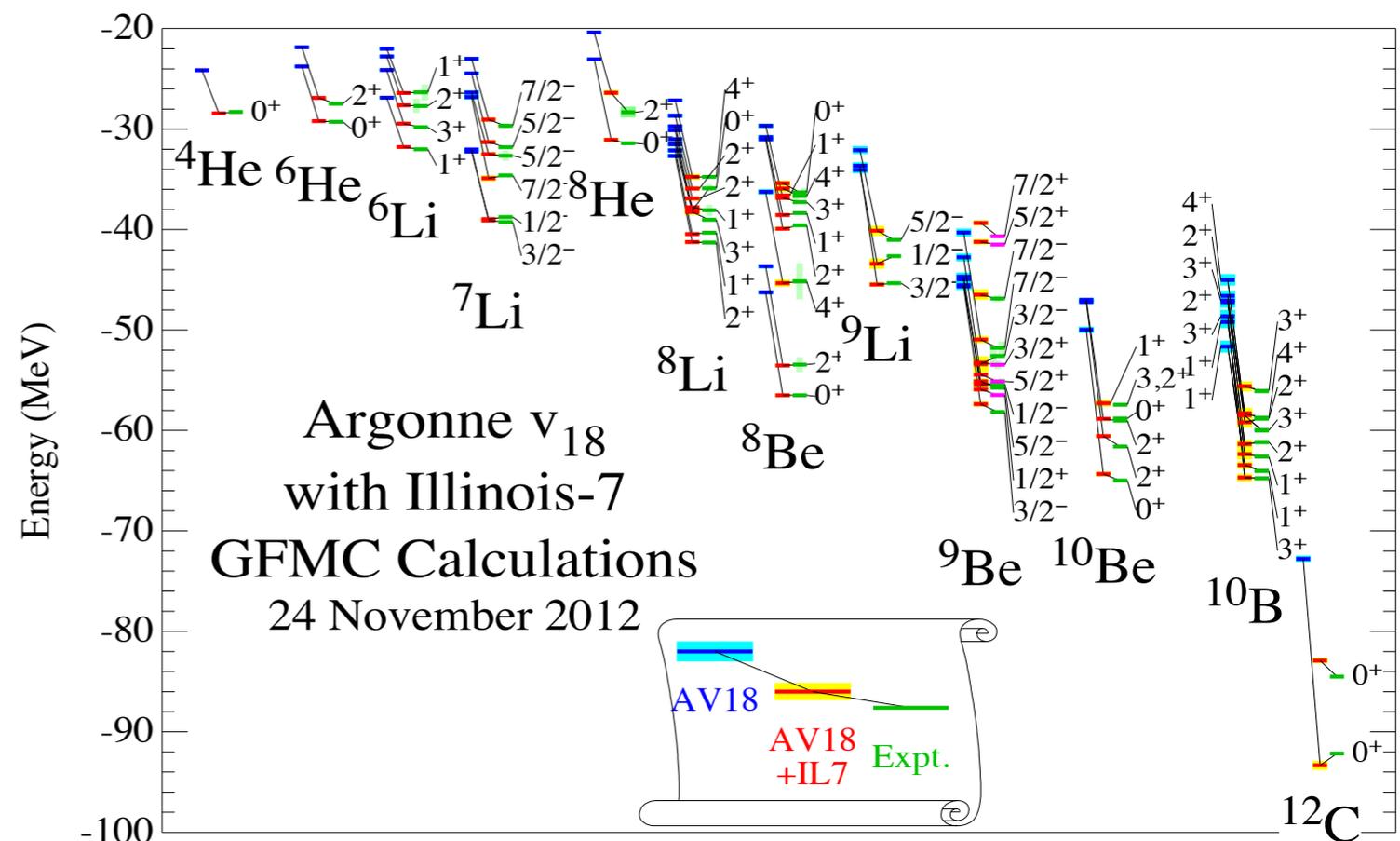
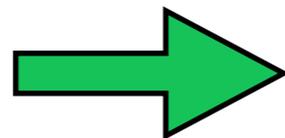
**Argonne v<sub>18</sub>:** 
$$v_{18}(r_{12}) = \sum_{p=1}^{18} v^p(r_{12}) \hat{O}_{12}^p$$

- To compute the sum rules and the longitudinal form factor, the ground state wave function of <sup>12</sup>C need to be precisely known.

Accurate 3-body potential: Illinois-7

+

Green's Function Monte Carlo



# Green's Function Monte Carlo

- Solving the many body Schroedinger equation is made particularly difficult by the complexity of the interaction, which is spin-isospin dependent and contains strong tensor terms

$$\hat{H}\Psi_0(x_1 \dots x_A) = E_0\Psi_0(x_1 \dots x_A) \quad \rightarrow$$

For  $^{12}\text{C}$  **270,336** second order coupled differential equations in 36 variables !!!

- GFMC algorithms use projection techniques to enhance the ground-state component of a starting trial wave function

$$\Psi_0(x_1 \dots x_A) = \lim_{\tau \rightarrow \infty} e^{-(\hat{H}-E_0)\tau} \Psi_T(x_1 \dots x_A)$$

- Sequence of imaginary time steps, each one consisting in a  $3A$  dimensional integral, evaluated within the Monte Carlo approach.

$${}_{\alpha} \langle \mathbf{r}'_1 \dots \mathbf{r}'_A | e^{-(\hat{H}-E_0)\Delta\tau} | \mathbf{r}_1 \dots \mathbf{r}_A \rangle_{\beta}$$

# Green Function Monte Carlo

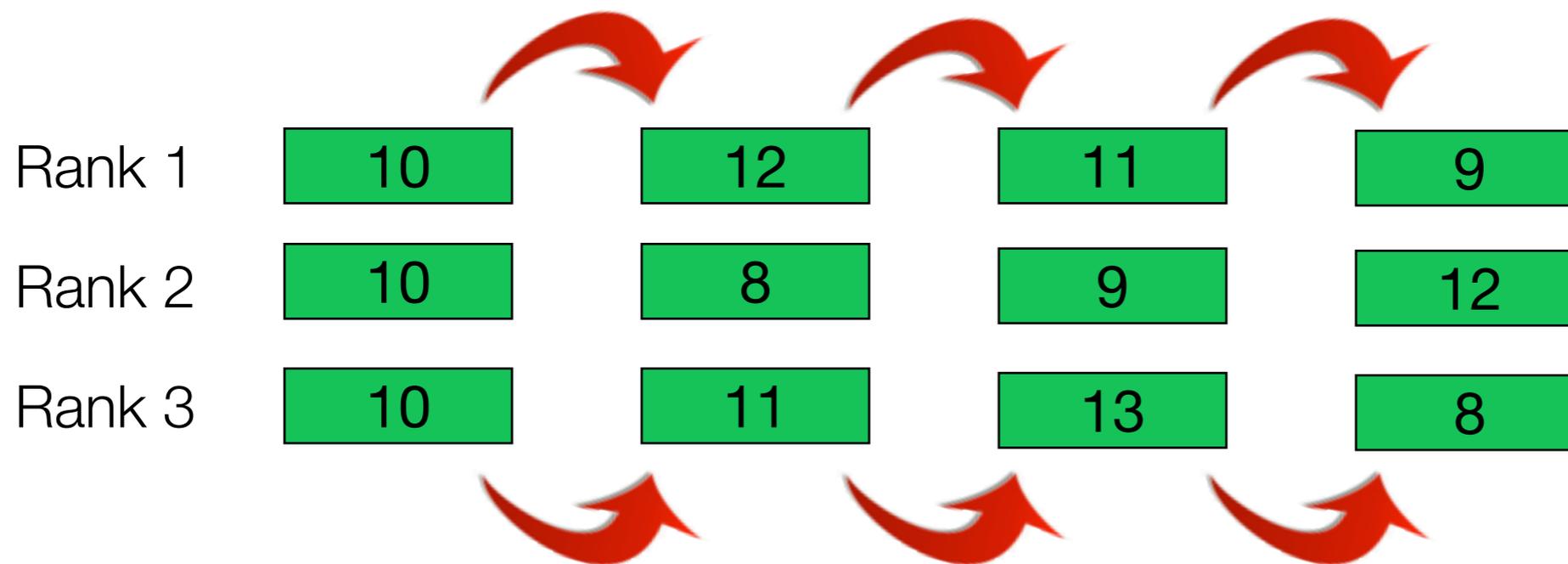
- Within GFMC the wave function is represented by a complex vector of  $2^A \binom{A}{Z}$  numbers, each depending on the 3A coordinates: **a GFMC sample**.
- The  ${}^3\text{H}$  case fits in the slide!

$$|\Psi_{3H}\rangle = \begin{pmatrix} a_{\uparrow\uparrow\uparrow} \\ a_{\uparrow\uparrow\downarrow} \\ a_{\uparrow\downarrow\uparrow} \\ a_{\uparrow\downarrow\downarrow} \\ a_{\downarrow\uparrow\uparrow} \\ a_{\downarrow\uparrow\downarrow} \\ a_{\downarrow\downarrow\uparrow} \\ a_{\downarrow\downarrow\downarrow} \end{pmatrix} \quad \hat{\sigma}_{12}|\Psi_{3H}\rangle = \begin{pmatrix} a_{\uparrow\uparrow\uparrow} \\ a_{\uparrow\uparrow\downarrow} \\ 2a_{\downarrow\uparrow\uparrow} - a_{\uparrow\downarrow\uparrow} \\ 2a_{\downarrow\uparrow\downarrow} - a_{\uparrow\downarrow\downarrow} \\ 2a_{\uparrow\downarrow\uparrow} - a_{\downarrow\uparrow\uparrow} \\ 2a_{\uparrow\downarrow\downarrow} - a_{\downarrow\uparrow\downarrow} \\ a_{\downarrow\downarrow\uparrow} \\ a_{\downarrow\downarrow\downarrow} \end{pmatrix}$$

# Need to go beyond MPI

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- The branching process of the GFMC algorithm involves replication and killing of the samples, the number of which can undergo large fluctuations.
- In the original version of the code, several Monte Carlo samples, say at least 10, were assigned to each rank.

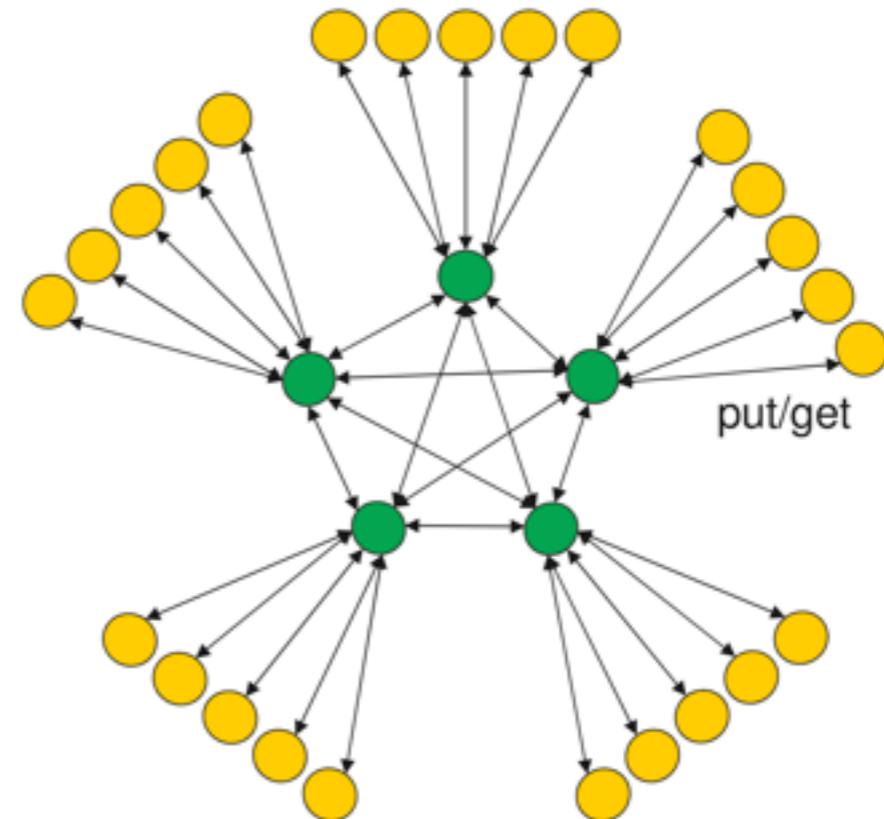
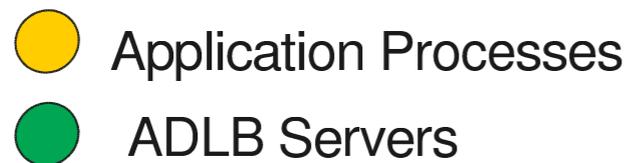


- A typical  $^{12}\text{C}$  calculation involves around 15,000 samples while leadership class computers have many 10,000's of processors, making the algorithm quite inefficient.

# ADLB library: overview

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- Nodes are organized in servers and slaves; in standard GFMC calculations approximately 3% of the nodes are ADLB servers.



- A shared work queue, managed by the servers, is accessed by the slaves that either put work units, denoted as “work packages” in it or get those work package out to work on them.
- Once a work package has been processed by a slave, a “response package” may be sent to the slave that put the work package in the queue.

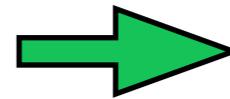
# ADLB library: implementation

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- In order to reduce the statistical error associated with GFMC, the sum rules and the longitudinal form factor are evaluated for:

**12** directions of the momentum transfer  
(in four groups of three orthogonal directions)

**21** values of the discretized momentum  
transfer magnitude



**252** independent  
expectation values  
need to be computed.

- The evaluation of the sum rules of the  $^{12}\text{C}$  for a single value of the momentum transfer takes of about 360 seconds (with 16 OMP threads)
- ADLB is used to split the calculation in such a way that each slave calculates the sum rules and the form factor for a single value of  $\mathbf{q}$ .

# ADLB library: implementation

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- The response work package contains the left and right wave functions and, in certain cases, their derivatives.

```
TYPE respon_wp_package_der
  sequence
! common part of package
  complex(8), dimension(nspin0, niso1) :: cfl, cfr
  complex(4), dimension(ns,niso1,3,npart0) :: cfdl, cfdr
  real(8) :: rpart0(3*npart0)
  real(8) :: actf, weight
  integer(4) :: iptb, if2, ijunk
  logical(4) :: prtsw
! variable part
  real(8), dimension(3) :: qh
  real(8) :: q
  integer(4) :: iqq, iqh
END TYPE respon_wp_package_der
```

As big as 1.30 GB !  
Impossible on Intrepid!

- ADLB solution

**Common part put:** called once for each configuration.

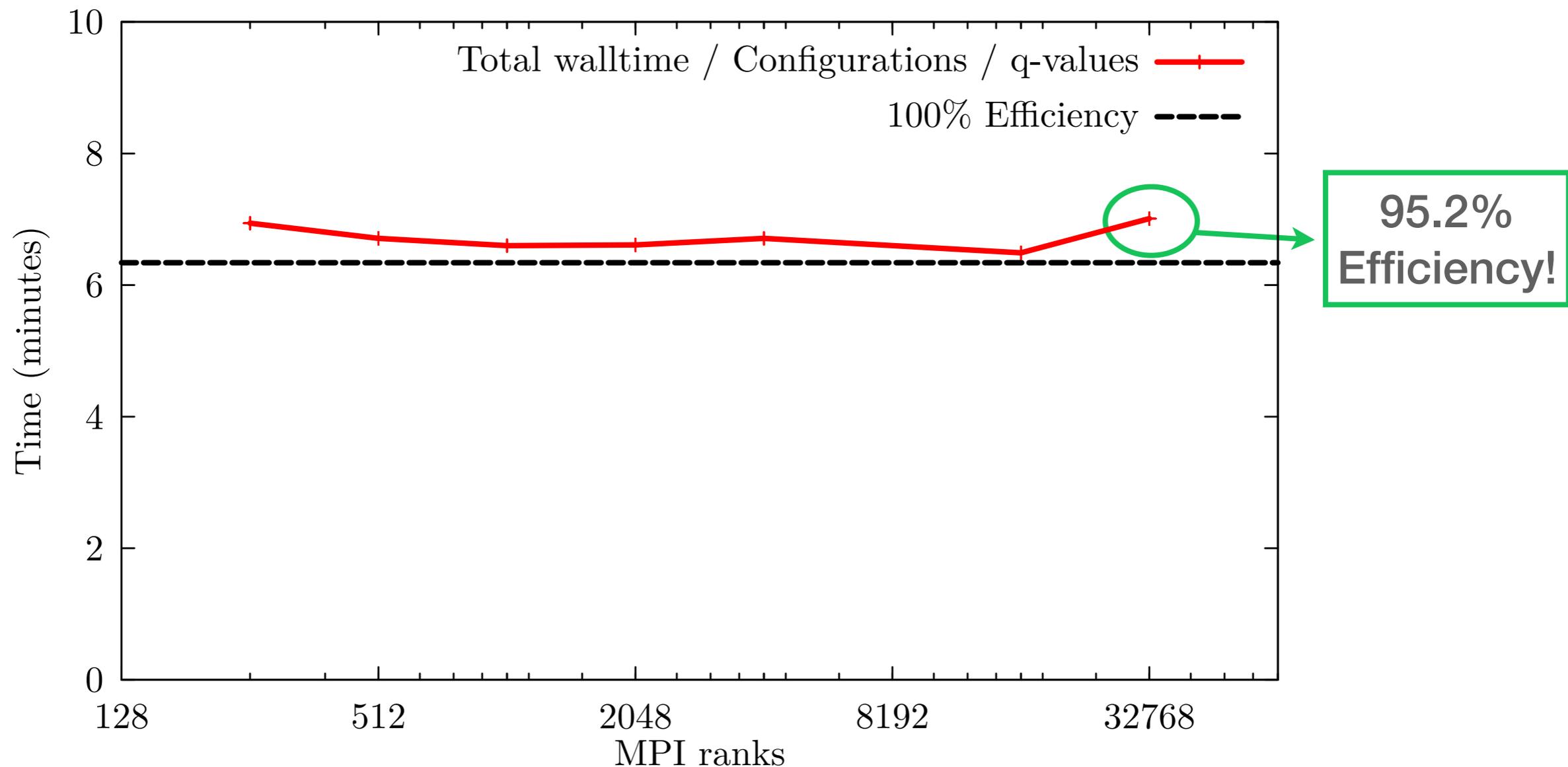
```
call ADLB_Begin_batch_put(rwp%cfl, respon_wp_len_common, ierr)
```

**Variable part put:** called for each **q**.

```
call ADLB_PUT(rwp%qh, respon_wp_len_var, -1, myid, adlbwp_respon, i_prior, ierr)
```

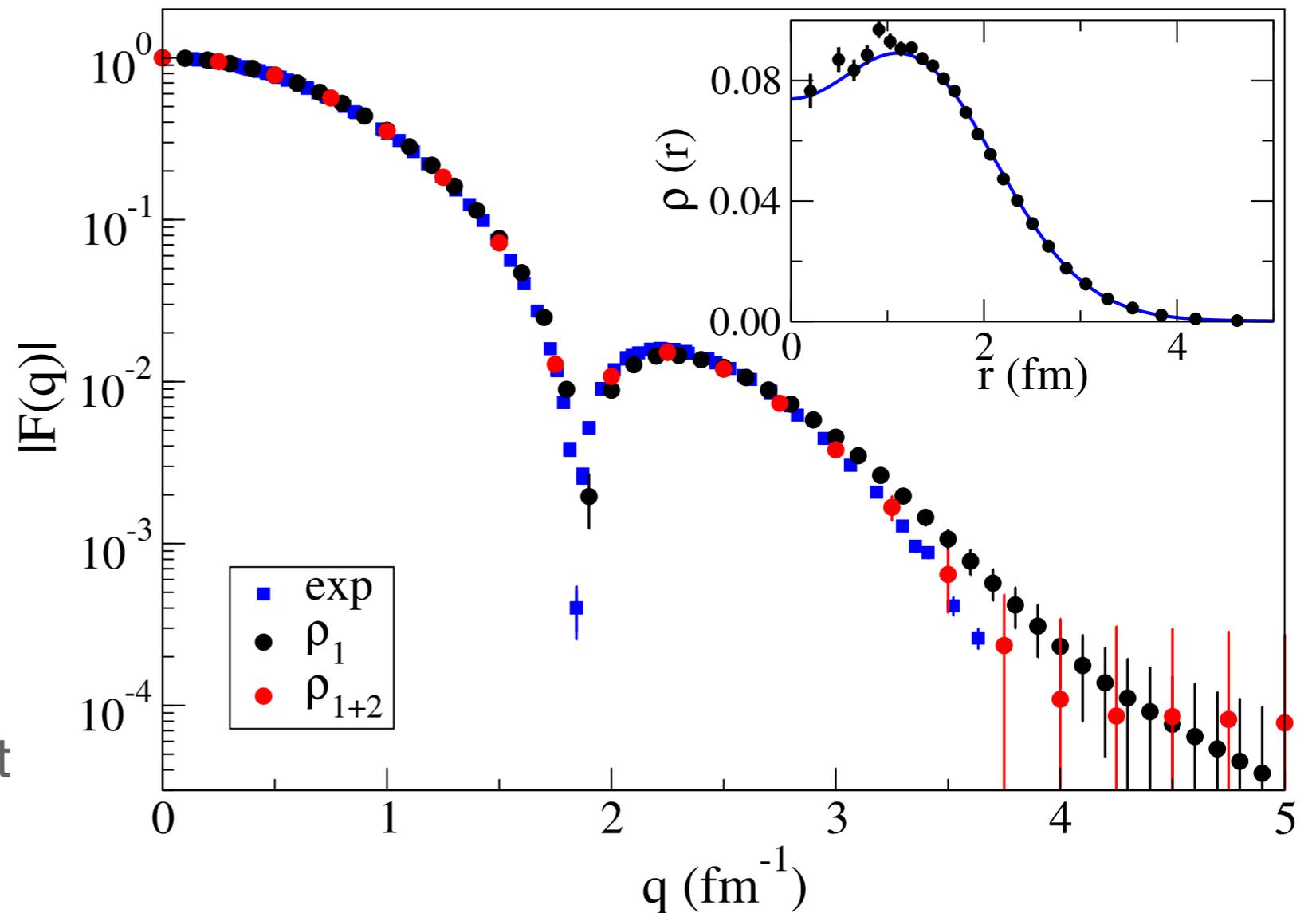
# ADLB library: performance

- Very good scaling of the calculation: total time per configuration per q-value very close to the ideal case.



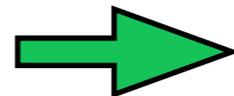
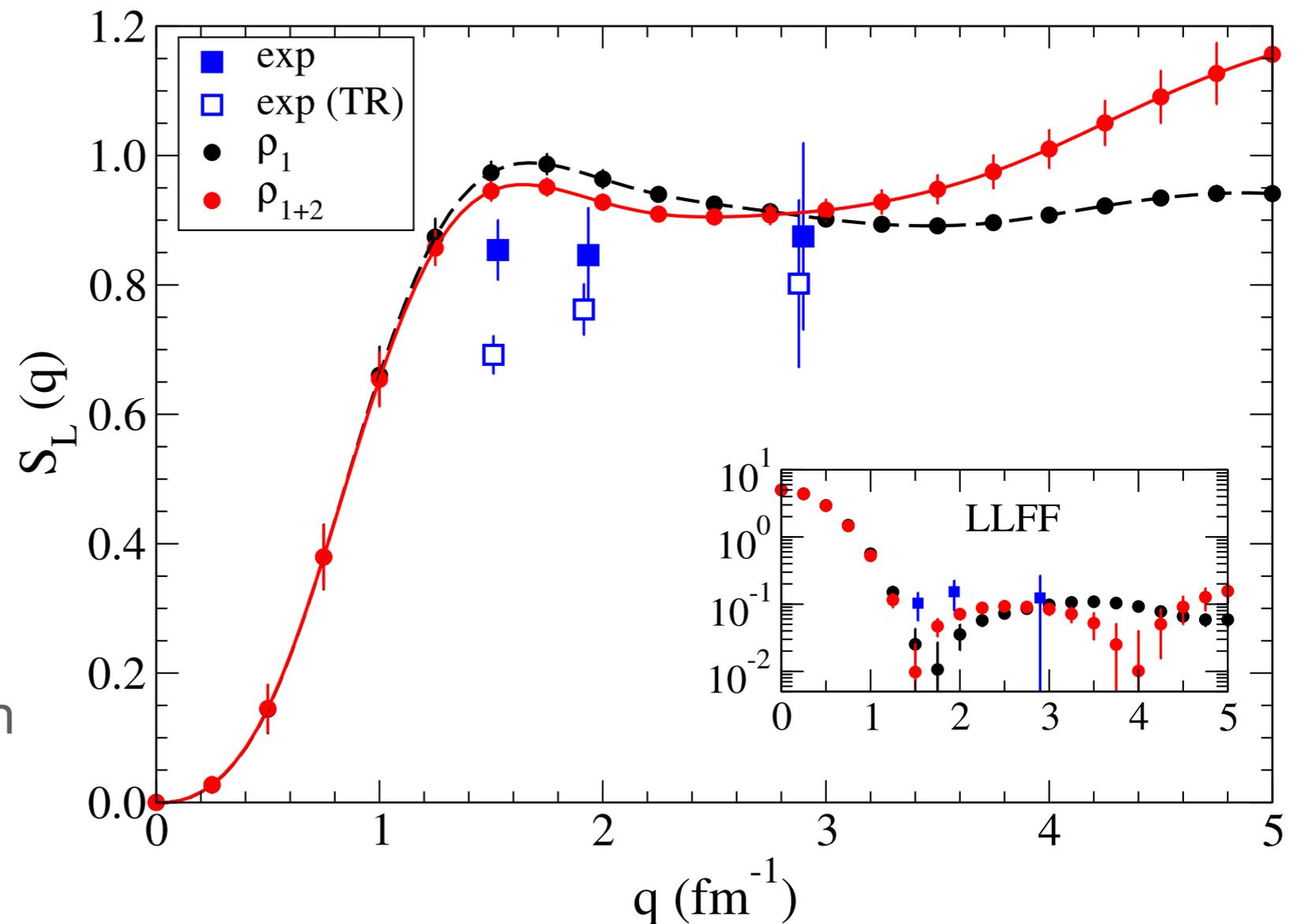
# Results - Longitudinal form factor

- Experimental data are well reproduced by theory over the whole range of momentum transfers
- Two-body terms become appreciable only for  $q > 3 \text{ fm}^{-1}$ , where they interfere destructively with the one-body contributions bringing theory into closer agreement with experiment.



# Results - Longitudinal sum rule

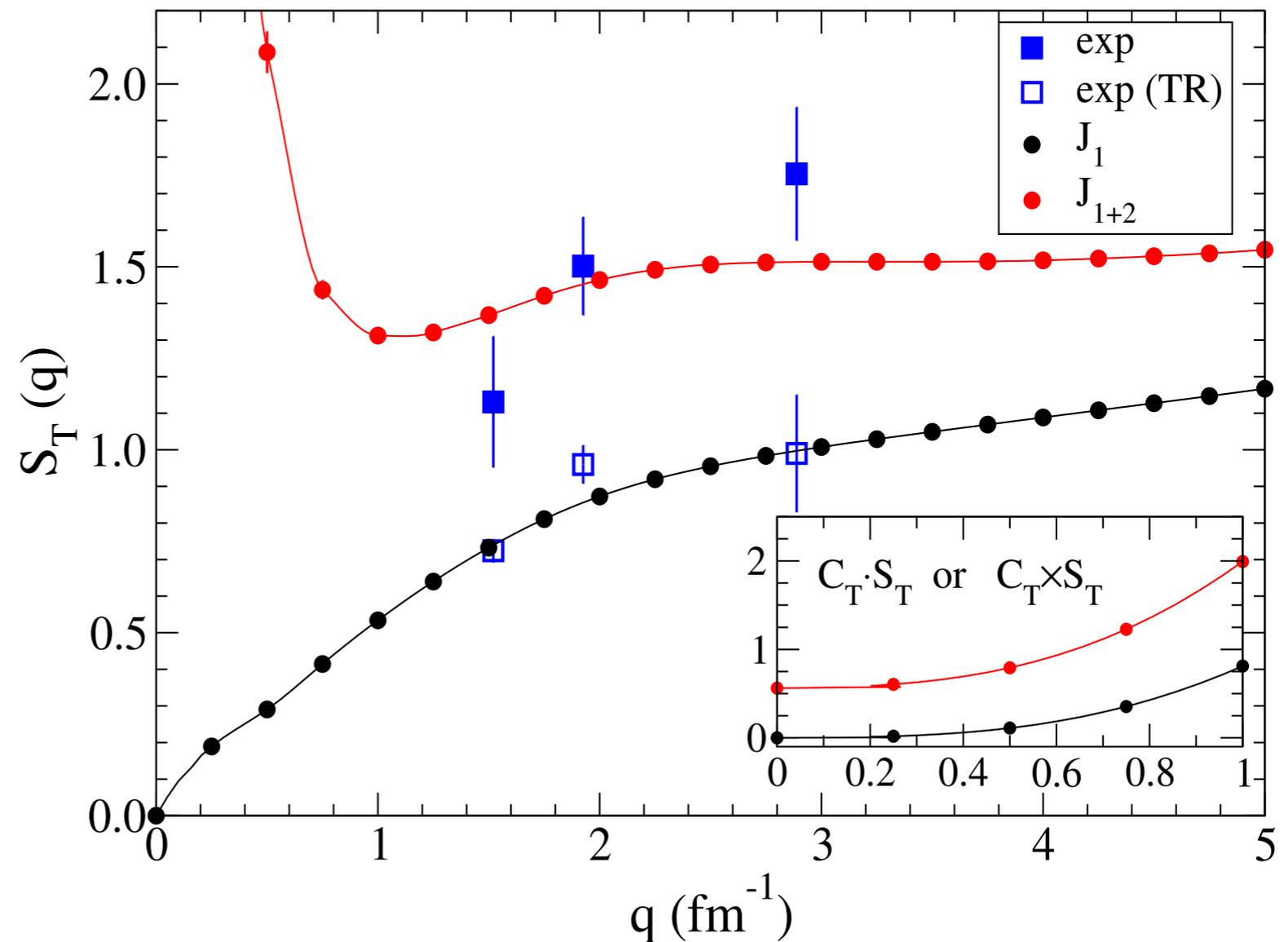
- $S_L$  vanishes quadratically at small momentum transfer.
- The one-body sum rule in the large  $q$  limit differs from unity because of relativistic correction and convection term.
- Satisfactory agreement with the experimental values, including tail contributions.
- No significant quenching of longitudinal strength is observed



No evidence for in-medium modifications of the nucleon electromagnetic form factors.

# Results - Transverse sum rule

- Divergent behavior at small  $q$  due to the normalization factor  $C_T$ .
- Comparison with experimental data made difficult by the  $\Delta$  peak.
- Large two-body contribution, most likely from the quasi-elastic region needed for a better agreement with experimental data.



# Conclusion & Future

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- Very good scaling to 32768 ranks (at least).
  - Very good description of the longitudinal form factor.
  - As for the sum rule (for the transverse in particular), comparison with experimental data is made difficult by the complexity of extracting the latter from the measured response functions.
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- The implementation of the Euclidean response calculation of  $^{12}\text{C}$  is currently under development. This will be more directly comparable with data.
  - Neutral current sum rules, allowing for the description of neutrino scattering on  $^{12}\text{C}$ , will be soon implemented in the code.