Spiral: Specialized FFTs

At ESSL and FFTW Speed

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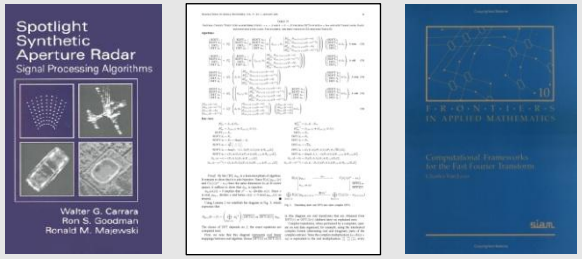
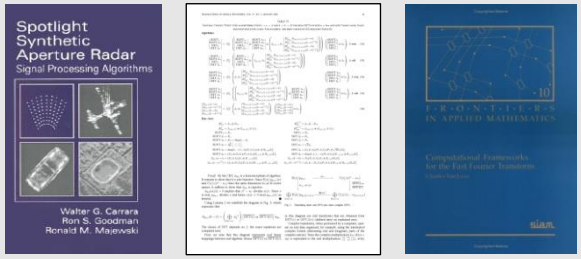
What is Spiral?

Traditionally



High performance library optimized for given platform

Spiral Approach



Spiral

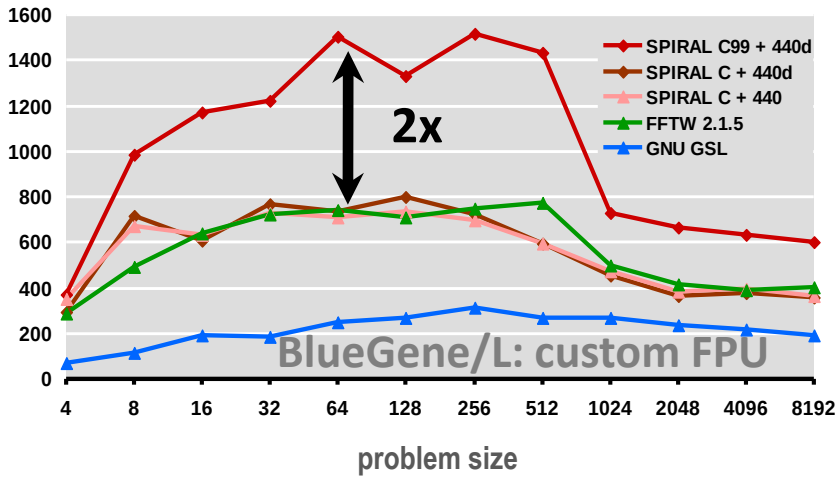
High performance library optimized for given platform

Comparable performance

BlueGene/L and P Node Performance

DFT, double precision, XL C compiler

performance [Mflop/s]



DFT, double precision, XL C compiler

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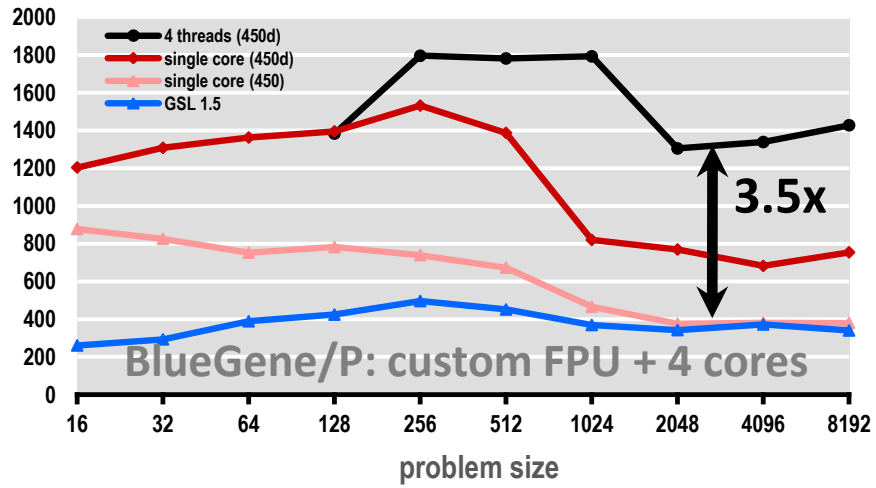


Single BlueGene/L CPU at 700 MHz
IBM T. J. Watson Research Center

SIMD vectorization

Single BlueGene/P node (4 CPUs) at 850 MHz
Argonne National Laboratory

SIMD vectorization + multi-threading

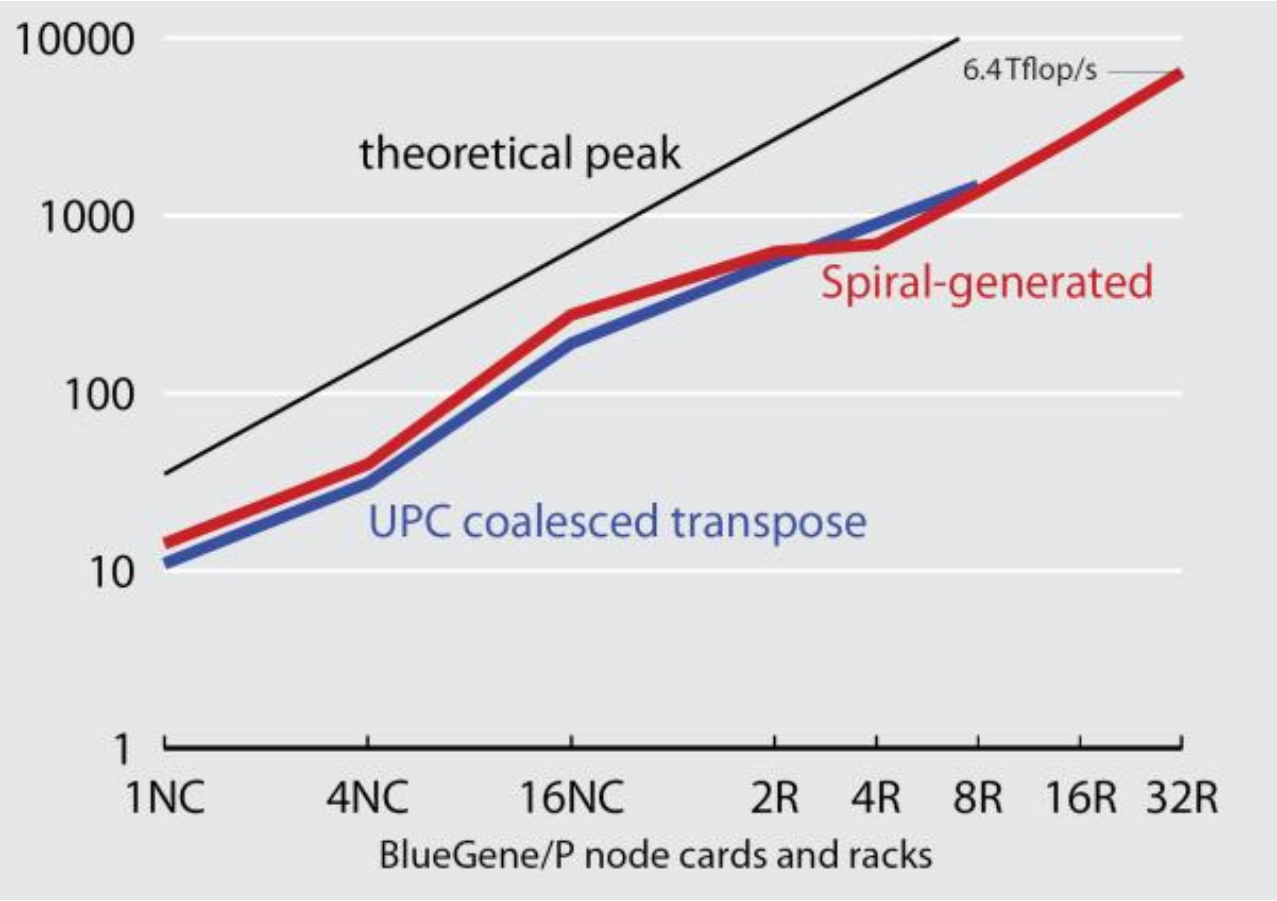
F. Gygi, E. W. Draeger, M. Schulz, B. R. de Supinski, J. A. Gunnels, V. Austel, J. C. Sexton, F. Franchetti, S. Kral, C. W. Ueberhuber, J. Lorenz: **Large-Scale Electronic Structure Calculations of High-Z Metals on the BlueGene/L Platform.** In Proceedings of Supercomputing, 2006. **Winner of the 2006 Gordon Bell Prize (Peak Performance Award).**

J. Lorenz, S. Kral, F. Franchetti, C. W. Ueberhuber: **Vectorization Techniques for the Blue Gene/L double FPU.** IBM Journal of Research and Development, Vol. 49, No. 2/3, 2005.

HPC Challenge Global FFT on BlueGene/P

1D Global FFT

performance [Gflop/s]

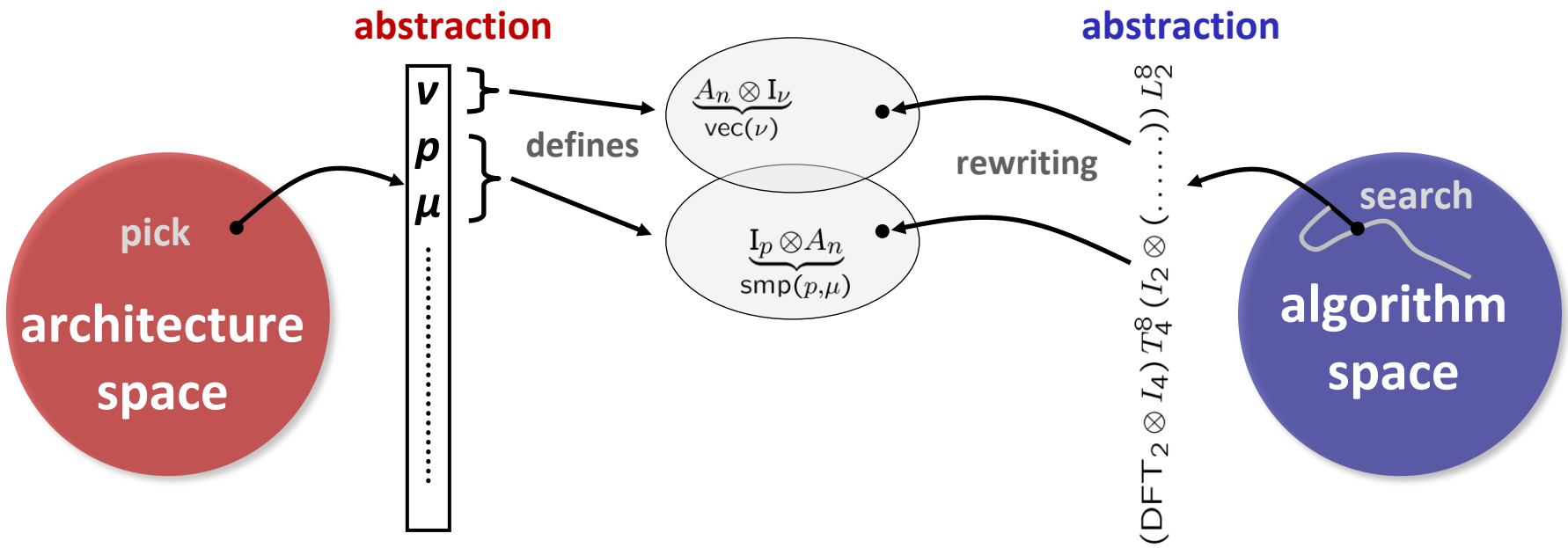


6.4 Tflop/s

G. Almási, B. Dalton, L. L. Hu, F. Franchetti, Y. Liu, A. Sidelnik, T. Spelce, I. G. Tānase, E. Tiotto, Y. Voronenko, X. Xue:
2010 IBM HPC Challenge Class II Submission. Winner of the 2010 HPC Challenge Class II Award (Most Productive System).

Spiral's Domain-Specific Program Synthesis

Model: common abstraction
= spaces of matching formulas



Architectural parameter:
Vector length,
#processors, …

optimization

Kernel:
problem size,
algorithm choice

Spiral's Formal Framework

- **Transform = Matrix-vector multiplication**

  Example: Discrete Fourier transform (DFT)

$$x \mapsto y = T \cdot x$$

input vector (signal)

output vector (signal)

transform = matrix

- **Fast algorithm = sparse matrix factorization = SPL formula**

  Example: Cooley-Tukey FFT algorithm

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} = \begin{bmatrix} 1 & \cdot & 1 & \cdot \\ \cdot & 1 & \cdot & 1 \\ 1 & \cdot & -1 & \cdot \\ \cdot & 1 & \cdot & -1 \end{bmatrix} \begin{bmatrix} 1 & \cdot & \cdot & \cdot \\ \cdot & 1 & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & j \end{bmatrix} \begin{bmatrix} 1 & 1 & \cdot & \cdot \\ 1 & -1 & \cdot & \cdot \\ \cdot & \cdot & 1 & 1 \\ \cdot & \cdot & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & \cdot & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot \\ \cdot & 1 & \cdot & \cdot \\ \cdot & \cdot & \cdot & 1 \end{bmatrix}$$

$$\mathrm{DFT}_4 = (\mathrm{DFT}_2 \otimes \mathrm{I}_2)\,\mathsf{T}^4_2\,(\mathrm{I}_2 \otimes \mathrm{DFT}_2)\,\mathsf{L}^4_2$$

Transforms and Breakdown Rules

$$\mathbf{DFT}_n \rightarrow (\mathbf{DFT}_k \otimes \mathrm{I}_m)\, \mathsf{T}^n_m (\mathrm{I}_k \otimes \mathbf{DFT}_m)\, \mathsf{L}^n_k, \quad n = km$$

$$\mathbf{DFT}_n \rightarrow P_n (\mathbf{DFT}_k \otimes \mathbf{DFT}_m) Q_n, \quad n = km,\ \gcd(k,m) = 1$$

$$\mathbf{DFT}_p \rightarrow R_p^T (\mathrm{I}_1 \oplus \mathbf{DFT}_{p-1}) D_p (\mathrm{I}_1 \oplus \mathbf{DFT}_{p-1}) R_p, \quad p \text{ prime}$$

DCT-3

DCT-4

"Teaches" Spiral about existing algorithm knowledge (~200 journal papers)

$$\mathbf{IMDCT}_{2m} \rightarrow (\mathrm{J}_m \oplus \mathrm{I}_m \oplus \mathrm{I}_m \oplus \mathrm{J}_m) \left(\left(\begin{bmatrix} 1 \\ -1 \end{bmatrix} \otimes \mathrm{I}_m \right) \oplus \left(\begin{bmatrix} -1 \\ -1 \end{bmatrix} \otimes \mathrm{I}_m \right) \right) \mathrm{J}_{2m}\, \mathbf{DCT\text{-}4}_{2m}$$

$$\mathbf{WHT}_{2^k} \rightarrow \prod_{i=1}^{t} (\mathrm{I}_{2^{k_1+\cdots+k_{i-1}}} \otimes \mathbf{WHT}_{2^{k_i}} \otimes \mathrm{I}_{2^{k_{i+1}+\cdots+k_t}}), \quad k = k_1 + \cdots + k_t$$

$$\mathbf{DFT}_2 \rightarrow \mathsf{F}_2$$

$$\mathbf{DCT\text{-}2}_2 \rightarrow \mathrm{diag}(1, 1/\sqrt{2})\, \mathsf{F}_2$$

$$\mathbf{DCT\text{-}4}_2 \rightarrow \mathrm{J}_2\, \mathsf{R}_{13\pi/8}$$

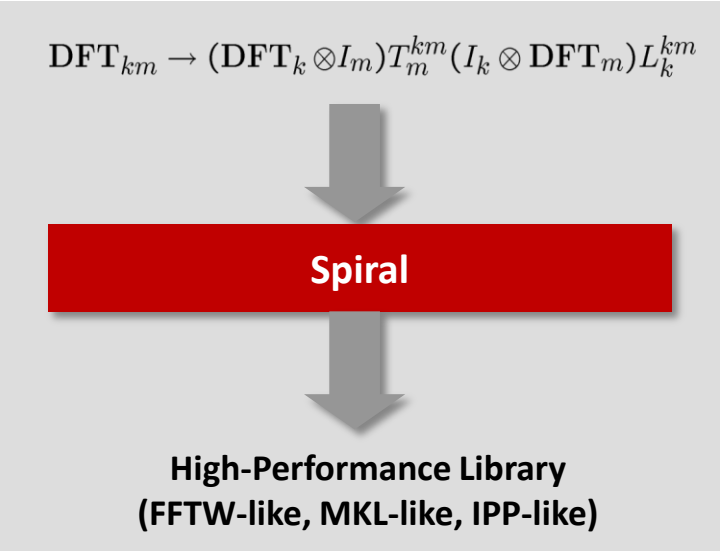
Base case rules

Teaches Spiral about FFT algorithms

Synthesizing FFTW From Its Specification

Input:

- **Transform**: DFT_n
- **Algorithms**: $\mathrm{DFT}_{km} \rightarrow (\mathrm{DFT}_k \otimes I_m) T_m^{km} (I_k \otimes \mathrm{DFT}_m) L_k^{km}$
  $\mathrm{DFT}_2 \rightarrow \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$
- **Vectorization**: 2-way SSE
- **Threading**: Yes



Output:

- Optimized library (10,000 lines of C++)
- For general input size
  (**not** collection of fixed sizes)
- Vectorized
- Multithreaded
- With runtime adaptation mechanism
- Performance competitive with hand-written code

Example Specialized FFTs Pruned FFT

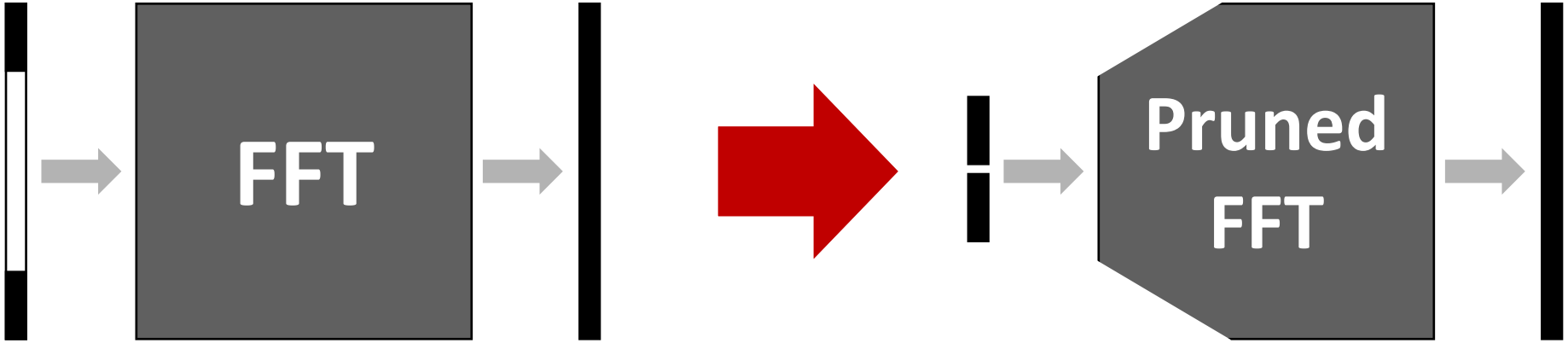
- **Recursive input pruning rule**

$$\mathrm{PDFT}_{kmn}^{\sigma \otimes km} \rightarrow (\mathrm{DFT}_m \otimes I_{kn}) T_{kn}^{kmn} L_m^{kmn} (\mathrm{PDFT}_{kn}^{\sigma \otimes k} \otimes I_m)$$

- **Base case**

$$\mathrm{PDFT}_n^{\sigma} \rightarrow \mathrm{DFT}_n \mathrm{S}_{\sigma}$$

- **Similar rule for output pruning and simultaneous pruning**



Spiral And BlueGene/Q

- **Single node library**
  fully optimized FFTW replacement

- **Multi-node replacement**
  generate optimized P3DFFT

- **Specialized FFTs for applications**
  work with HACC and PS/DNS

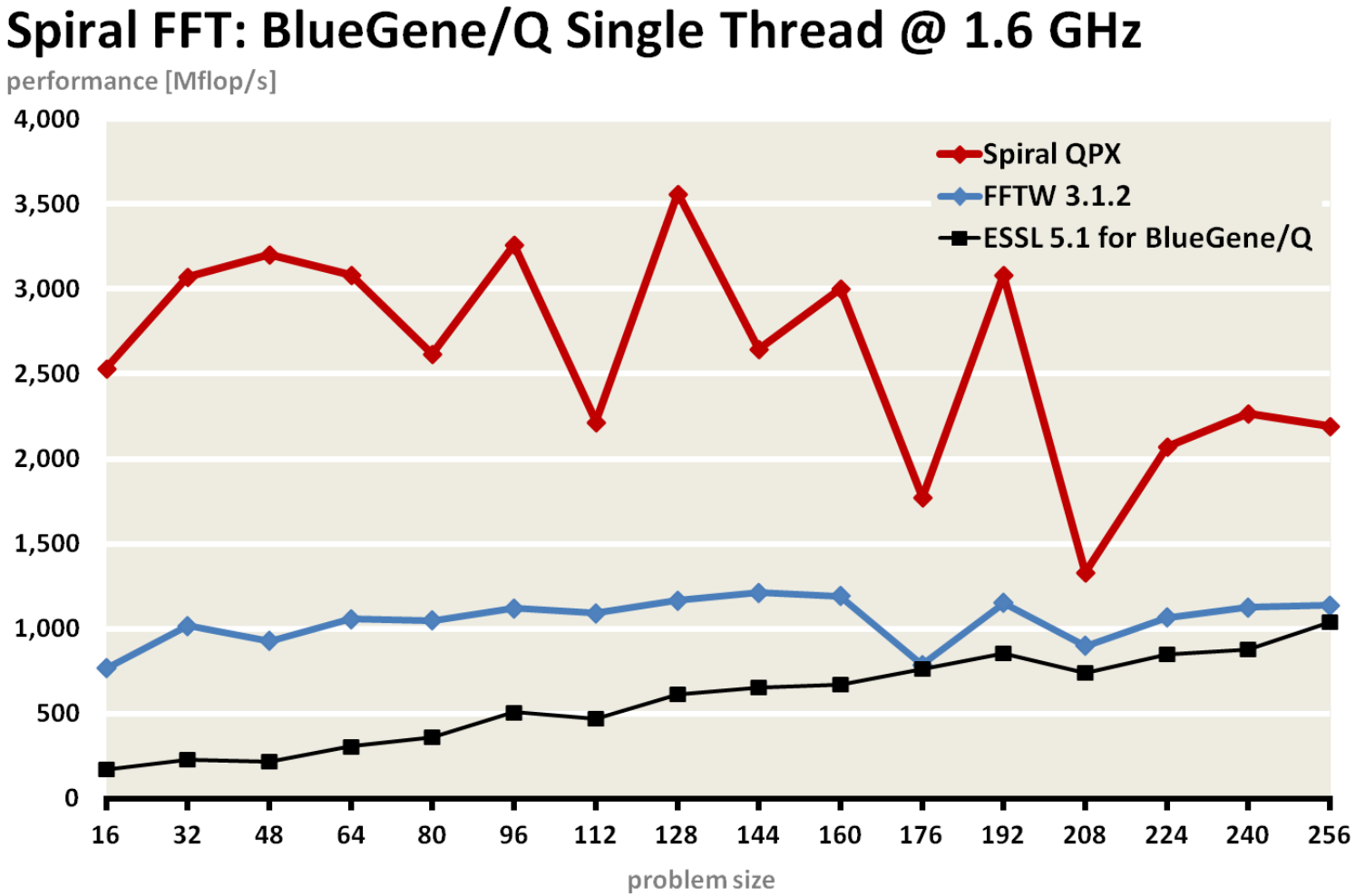
- **Higher-level FFT-heavy kernels**
  convolution, correlation, Poisson solvers

The Fun Just Started…

```
void dft16(double  *Y, double  *X) {
    const vector4double C1 = (vector4double)(1.0, 0.70710678118654757, 0.0, (-0.70710678118654757));
    const vector4double C2 = (vector4double)(0.0, 0.70710678118654757, 1.0, 0.70710678118654757);
    const vector4double C3 = (vector4double)(1.0, 0.92387953251128674, 0.70710678118654757, 0.38268343236508978);
    const vector4double C4 = (vector4double)(0.0, 0.38268343236508978, 0.70710678118654757, 0.92387953251128674);
    const vector4double C5 = (vector4double)(1.0, 0.38268343236508978, (-0.70710678118654757), (-0.92387953251128674));
    const vector4double C6 = (vector4double)(0.0, 0.92387953251128674, 0.70710678118654757, (-0.38268343236508978));
    vector4double a90, a91, a92, a93, a94, a95, s139, s140, s141, s142, s143, s144, s145, s146, s147, s148,...,;
    vector4double  *a89, *a96;
    a89 = ((vector4double  *) X);
    s139 = a89[0];
    s140 = a89[1];
    a90 = vec_gpci(0xa60);
    s141 = vec_perm(s139, s140, a90);
    a91 = vec_gpci(0xef2);
    s142 = vec_perm(s139, s140, a91);
    s143 = a89[4];
    s144 = a89[5];
    s145 = vec_perm(s143, s144, a90);
    ...
    s170 = vec_perm(s158, s162, a95);
    s171 = vec_sub(vec_mul(C1, s165), vec_mul(
    s172 = vec_add(vec_mul(C2, s165), vec_mul(
    t145 = vec_add(s163, s171);
    t146 = vec_add(s167, s172);
    t147 = vec_sub(s163, s171);
    t148 = vec_sub(s167, s172);
    s173 = vec_sub(vec_mul(C3, s164), vec_mul(
    s174 = vec_add(vec_mul(C4, s164), vec_mul(
    s175 = vec_sub(vec_mul(C5, s166), vec_mul(
    s176 = vec_add(vec_mul(C6, s166), vec_mul(
    t149 = vec_add(s173, s175);
    ...
    a96[3] = s182;
    s183 = vec_perm(t159, t160, a92);
    a96[6] = s183;
    s184 = vec_perm(t159, t160, a93);
    a96[7] = s184;
}
```

```
78| 00014C qvfmul   118D0132   1     QVFMUL    qr12=qr13,qr4,fcr
79| 000150 qvfmul   11AD0172   1     QVFMUL    qr13=qr13,qr5,fcr
84| 000154 qvfmul   11CF01B2   1     QVFMUL    qr14=qr15,qr6,fcr
85| 000158 qvfmul   11EF01F2   1     QVFMUL    qr15=qr15,qr7,fcr
86| 00015C qvfmul   12110232   1     QVFMUL    qr16=qr17,qr8,fcr
87| 000160 qvfmul   12310272   1     QVFMUL    qr17=qr17,qr9,fcr
60| 000164 qvfperm  1253B00C   1     QVFPERM   qr18=qr19,qr22,qr0
62| 000168 qvfperm  1273B04C   1     QVFPERM   qr19=qr19,qr22,qr1
65| 00016C qvfperm  12F4A80C   1     QVFPERM   qr23=qr20,qr21,qr0
66| 000170 qvfperm  1294A84C   1     QVFPERM   qr20=qr20,qr21,qr1
72| 000174 qvfperm  12B2B8CC   1     QVFPERM   qr21=qr18,qr23,qr3
73| 000178 qvfperm  12D3A08C   1     QVFPERM   qr22=qr19,qr20,qr2
74| 00017C qvfperm  1073A0CC   1     QVFPERM   qr3=qr19,qr20,qr3
79| 000180 qvfnmadd 10B62B3E   1     QVFNMADD  qr5=qr5,qr22,qr12,fcr
80| 000184 qvfmadd  1096237A   1     QVFMADD   qr4=qr4,qr22,qr13,fcr
85| 000188 qvfnmadd 10F53BBE   1     QVFNMADD  qr7=qr7,qr21,qr14,fcr
86| 00018C qvfmadd  10D533FA   1     QVFMADD   qr6=qr6,qr21,qr15,fcr
87| 000190 qvfnmadd 11234C3E   1     QVFNMADD  qr9=qr9,qr3,qr16,fcr
88| 000194 qvfmadd  1063447A   1     QVFMADD   qr3=qr8,qr3,qr17,fcr
70| 000198 qvfperm  1112B88C   1     QVFPERM   qr8=qr18,qr23,qr2
75| 00019C qvfperm  104A588C   1     QVFPERM   qr2=qr10,qr11,qr2
81| 0001A0 qvfadd   1148282A   1     QVFADD    qr10=qr8,qr5,fcr
82| 0001A4 qvfadd   1162202A   1     QVFADD    qr11=qr2,qr4,fcr
89| 0001A8 qvfadd   1187482A   1     QVFADD    qr12=qr7,qr9,fcr
90| 0001AC qvfadd   11A6182A   1     QVFADD    qr13=qr6,qr3,fcr
83| 0001B0 qvfsub   10A82828   1     QVFSUB    qr5=qr8,qr5,fcr
```

First Performance Results



More Information:

www.spiral.net

www.spiralgen.com